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Unstablity of a punishment strategy in correlated equilibria

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Abstract— In game theory, to realize correlated equilibria without a trusted meditator, the idea of replacing a mediator with protocol execution by players is suggested. Before players take actions in a game, players communicate each other by following a protocol. In that model, the concept of a punishment strategy is defined for cases that a player (or some players) aborts the protocol. In this paper, we show an example of game in which a punishment strategy does not work and suggest an improved definition of a punishment strategy.

Keywords: Game theory, Nash equilibrium, Correlated equibrium, Punishment strategy

1 Introduction

For years, in the field of cryptography, researchers concern about applying game theory to cryptography. This is because cryptography and game theory are concerned with the study of interactions among mutually distrusting players. Cryptographic protocols are designed under the assumption that some players are honest and faithfully follow the protocol, while some players are malicious and behave arbitrarily. However in game theory, all players are only considered to be rational and behave in order to maximize their profits. In traditional cryptography theory, if a player is corrupted, it is considered to be dishonest and to even take an unreasonable action that the other players can not expect. However in game theory, almost as same as real world, it is assumed that each player selects its action in view of the profit it can receive even if it is not honest.

One of the most important ideas in game theory is equilibrium in which it is the best way for all player to follow actions. Two kinds of equilibrium were proposed. First, Nash equilibrium (named after John Forbes Nash, who proposed it) is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by only changing its own strategy([E94]). The other is a correlated equilibrium which was proposed by Robert Aumann [A74] and is a solution concept that is more general than the well known Nash equilibrium. The idea is that each player chooses its action according to its observation of the value of the single public signal. That signal is supposed to be sent by a third trusted party called a mediator. It chooses the set of moves according to the right joint distribution and privately tells each player what its designated move is. Then the next question is "can we remove the mediator by using some protocols". In the case of a two-player game, it is well known that in the standard cryptographic models the answer is positive, provided that the two players can interact(see [GMW87]), This positive result can be carried over to the game theory model as well. Specially, we consider an extended game, in which the players first exchange some messages (this part is called "cheap talk" in game theory), and then choose their actions and execute them simultaneously as in the original game. In [DHR00], they suggested the concept of punishment strategy which is a kind of rule for players not to abort in the cheap talk phase. If a player aborts, the other players take actions that lead aborting player's utility low. So all player do not have incentive to abort in the cheap talk phase and deviating from the action in the original game. In this paper, we show an example of game in which a punishment strategy does not work and suggest an improved definition of a punishment strategy.

2 Preliminaries

2.1 Game theory

In game theory we assume players take actions and have their own utility functions that is determined by a set of all players' actions. An n-player game Γ is denoted by

$\Gamma = (\{A_i\}_{i=1}^n, \{u_i\}_{i=1}^n)$.

A_i is a set of actions of each player i(P_i from now on). Player P_i selects an action $a_i \in A_i$. u_i is a utility function of P_i. $N = \{P_1, P_2, \ldots P_n\}$ is the set of all players. The game is played by having every player takes action $a_i \in A_i$ simultaneously. The payoff to P_i is given by $u_i(\boldsymbol{a})$, where $\boldsymbol{a}$ is the tuple of each player's action ($\boldsymbol{a} = (a_i, \ldots, a_n)$). P_i prefers outcome $\boldsymbol{a}$ to outcome $\acute{\boldsymbol{a}}$ iff $u_i(\boldsymbol{a}) \geq u_i(\acute{\boldsymbol{a}})$. We say P_i strictly prefers outcome α to outcome $\acute{\alpha}$ if $u_i(\alpha) > u_i(\acute{\alpha})$ and P_i weakly prefers α to α if $u_i(\alpha) \geq u_i(\acute{\alpha})$. We assume that information of all players' possible actions A=$A_1 \times \ldots \times A_n$ and util-

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		P_2		
		$\acute{A}$	$\acute{B}$	$\acute{C}$
P_1	A	(11,6)	(7,8)	(8,10)
	B	(8,6)	(10,10)	(6,7)
	C	(8,12)	(4,3)	(10,9)

Figure 1: Two player game.

ity functions $u = u_1 \times \ldots \times u_n$ are common knowledge among the players.

We show an example of two-player game in Figure 1. It can be represented in a matrix form by labeling actions of A_1 to rows and A_2 to columns.

The entry in the cell at row $a_1 \in A_i$ and column $a_2 \in A_2$ contains a tuple (u_1,u_2) indicating the payoffs to P_1 and P_2, respectively, given the outcome $\boldsymbol{a} = (a_1,a_2)$. The example in Fig.1 represents a game where $A_1 = \{A, B, C\}$, $A_2 = \{\acute{A}, \acute{B}, \acute{C}\}$, and e.g., $u_1(A, \acute{A}) = 11$ and $u_2(A, \acute{A}) = 6$.

2.2 Nash equilibrium

If players play a game and P_1 knows the actions the other players will take, P_1 will select an action $a_1 \in A_1$ that maximizes $u_1(\boldsymbol{a})$. If a_1 is the best way a_1 is called a best response of P_1 to the actions of the other players. If for every player's action a_i is the best response to the other actions, we call the tuple of actions ($\boldsymbol{a} = (a_1, ..., a_n) \in A$) a Nash equilibrium. Formally, we define $\boldsymbol{a_{-i}} = (a_1, .., a_{i-1}, a_{i+1}, .., a_n)$ and let $(\acute{a_i}, \boldsymbol{a_{-i}})$ denote $(a_1, ..., a_{i-1}, \acute{a_i}, a_{i+1}, ..., a_n)$.

In a Nash equilibrium each player can not receive an additional profit by deviating its strategy. In the example in Fig. 1, P_1 may think that P_2 select $\acute{A}$ to receive the maximum payoff 12 ((a_1,a_2) = (C,$\acute{A}$)), so P_1 may select strategy A to receive the maximum payoff 11 under the assumption that P_2 will take $\acute{A}$. However, if P_2 thinks that P_1 takes this strategy, $\acute{C}$ becomes a better strategy for P_2.
$u_1(A, \acute{B}) \leq u_1(B, \acute{B}) \geq u_1(C, \acute{B})$
$u_2(B, \acute{A}) \leq u_2(B, \acute{B}) \geq u_2(B, \acute{C})$
So B is the best response to actions of P_2 and $\acute{B}$ is the best response to actions of P_1. In this case, the set of actions $(B, \acute{B})$ fulfills the condition of a Nash equilibrium $u_i(\acute{a_i}, \boldsymbol{a_{-i}}) \leq u_i(\boldsymbol{a})$ for all i.

2.3 Correlated equilibrium

The concept of correlated equilibrium is suggested in [A74]. It may give a better payoff than Nash equilibrium for every player P_i. A correlated equilibrium can be described by means of a joint distribution over the strategy sets.

Let $\Gamma = (\{A_i\}_{i=1}^n, \{u_i\}_{i=1}^n)$ be an n-player game. $\alpha \in A_1 \times \ldots \times A_n$ denotes the set of n-tuple strategies of Γ. We assume the existence of external party M called the mediator and define a mediated version of Γ which relies on M.

The game is now played in two stages: first, the mediator chooses a tuple of actions $\boldsymbol{a} = (a_1, \ldots, a_n) \in A$ according to some known distribution D, and then hands the recommendation a_i to player P_i. The players then play Γ as before by choosing any action in their respective action sets. Players are supposed to follow the recommendation of M, and it is the best response for each player to realize a correlated equilibrium. To formally define this notion, let $u_i(\acute{a_i}, \boldsymbol{a_{-i}}|a_i)$ denote the expected utility of P_i, given that it plays action $\acute{a_i}$ after having received recommendation a_i and all other players play their recommended actions $\boldsymbol{a_{-i}}$.

Definition 1 *Let $\Gamma = (A_i, u_i)$. A distribution $D \in \Delta(A)$ is a correlated equilibrium if for all $\boldsymbol{a} = (a_1, \ldots, a_n)$ in the support of D, all i, and all $\acute{a_i} \in A_i$, it holds that* $u_i(\acute{a_i}, \boldsymbol{a_{-i}}|a_i) \leq u_i(\boldsymbol{a}|a_i)$.

2.4 Realizing correlated equilibrium with cheap talk

Consider some n-player game $\Gamma = (A_i,u_i)$ in normal form, along with a correlated equilibrium D. We then define the extensive form game Γ_{CT} in which all players first communicate in a cheap talk phase before the original game Γ. Following the game-theoretic convention, all players must play some actions in Γ (i.e., we do not allow player P_i to abort in Γ unless this is an action in A_i). On the other hand, following the cryptographic convention we allow players to abort during the cheap talk phase. In case players abort in the cheap talk phase, we have to consider new idea for each player to move properly.

3 Punishment strategy

Punishment strategy was suggested as a kind of rules that prevents players from aborting in the cheap talk phase. If a player aborts, the other players take actions that make aborting player's utility low. So all player do not have incentive to abort in the cheap talk phase and deviating from an action in the original game. The initial result of punishment strategy was shown in [DHR00], that examines the case of two-player game. The basic idea is as follows: Let D be a correlated equilibrium in a two-player games Γ in Γ_{CT}, the two players run a protocol Π to calculate $(a_1, a_2) \leftarrow D$, where player P_i receives a_i as an output. This protocol Π is secure-with-abort (cf.[G04]), which informally means that privacy and correctness holds, on the other hand, fairness does not; in particular, we assume it is possible for P_1 to receive its output even though P_2 does not. After running Π, each player plays the action it received as the output in Π; if P_2 does not receive an output from Π then it plays the minimax profile against P_1. The minimax profile against P_i is an action $a_{-i} \in A_{-i}$ that minimizes $max_{a_i \in A_i}\ u_i(a_i, a_{-i})$. Katz generalized this punishment strategy from two-player to n-player in [K08]. Assume that some players select actions following the recommendation from the outputs of Π, while some collude with each other (which is called coalition C) and deviate from recommendation. C prefers σ to $\acute{\sigma}$ only if every player in

C weakly prefers σ to $\acute{\sigma}$ and some player in C strictly prefers σ to $\acute{\sigma}$.

Definition 2 *Let Γ be an n-player game with correlated equilibrium D. A strategy vector ρ is a t-punishment strategy with respect to D if for all $C \subseteq N$ with $\mid C \mid \leq t$, and all $\acute{\sigma}_C$ it holds that for all $i \in C$ $u_i(\acute{\sigma}_C, \rho_{-C}) \leq u_i(D)$.*

We introduce another definition of punishment strategy in [ADH08]. In [ADH08] they considered the case with k-immune which tolerates to Byzantine failure players (If there is nothing that players in a set T of size at most k can do to give the rest of players a worse payoff, even if the players in T can communicate with each other). For simplicity of discussion, this paper assumes that k=0, that is, there is no Byzantine failure players. They also consider type t_i which is an input given to each player at the beginning. This paper does not consider type t_i, that is, there is a single type for every player. The example in this paper can be easily extended to the cases where there are multiple types for players.

Definition 3 *If Γ is an underlying game with a mediator M, a strategy profile ρ in Γ is a t-punishment if for all subsets $C \subseteq N$ with $\mid C \mid \leq t$, all strategies σ in Γ with a cheap talk CT(C) among players in C, and all players $i \in C$ $u_i(\Gamma, \sigma) > u_i(\Gamma + CT(C), \sigma_C, \rho_{-C})$*

A remarkable difference between Definition 2 and Definition 3 is allowing equal utilities or not. In this meaning, Definition 3. requires the stronger condition. Intuitively, for any set C, even if all players in C collude and communicate each other with the cheap talk, no player in C can obtain a better payoff than the correlated equilibrium if the rest of the players select the punishment strategy. In [ADGH06], they showed that for six-player games with a 2-punishment strategy, any Nash equilibrium can be implemented even in the presence of at most one malicious player.

4 Cheating player's action against punishment strategy

This section shows an example that the punishment strategy does not prevent the players in C aborting in the cheap talk phase. $N = \{P_1, P_2, P_3, P_4, P_5\}, t = 2$, $A_i = \{a_i^1, a_i^2, a_i^3, a_i^4\}$ for $1 \leq i \leq 5$. The utility u_i is shown in Figure 2 and 3. Let us consider the case when P_1 and P_2 abort in the cheap talk phase. The punishment strategy for $C = (P_1, P_2)$ is (a_3^4, a_4^4, a_5^5). After aborting the protocol, they declare that they will take actions a_1^1 and a_2^2, the rest of players are supposed to select the punishment strategy $(a_4^3$,a_4^4,$a_4^5)$ and each player will receive a payoff $(u_1$,u_2,u_3,u_4,$u_5) = (4,4,4,4,4)$. In game theory, all players are considered to be rational, so if there are better a set of actions for P_3, P_4, and P_5, it is natural for them to select better actions than Nash equilibrium. The rest of players P_3, P_4, and P_5 know the actions which P_1 and P_2 take. They know P_1 and P_2 are rational and P_1 and P_2 know they are rational, The utility for (P_3, P_4, P_5) of the punishment strategy is worse than that of the other strategy. If the players are honest, they will select a punishment strategy even if they receive worse payoff than the other strategies. However the players are rational and all players know they are rational. Thus the aborting players thinks they will not execute the punishment strategy. This is called as "empty threat" [DHR00].

In this example, in P_3's view, a_3^1 is the dominant strategy given that P_1 and P_2 take a_1^1, a_2^2. Thus P_4 and P_5 will think P_3 take action a_3^1. And given a set of actions $(a_1^1$,a_2^2,$a_3^1)$, a_4^3 is the dominant strategy for P_4. So the last player P_5 have to select a_5 to receive a maximum profit under assumption that all players are rational. P_5 is supposed to select an action a_5^1. So, $(a_1^1$, a_2^2,a_3^1,a_4^3,$a_5^1)$ is the equilibrium for the all players. And in this case, even if P_1 and P_2 abort in the cheap talk phase, the other players will not punish them, rather help them for receiving more payoff to get more payoff than the punishment strategy.

5 Observation

The reason a punishment strategy does not work is that definitions in [K08] and [ADH08] do not care about punishing players' utilities. In [DHR00], it was shown that for two-player games, a min-max strategy may be "empty threat" without proper setting. For multiple player games, the above example shows that a punishment strategy does not work. To avoid the cases shown above, we suggest new definition of a punishment strategy which considers punishing players' utilities.

Definition 4 *Let Γ be an n-player game with correlated equilibrium D. A strategy vector ρ is a t-punishment strategy if for any $C \subseteq N$ such that $\mid C \mid \leq t$, any strategy vector $\acute{\sigma}_C$ for C, and any strategy vector $\acute{\rho}$ for $-C$, it holds that for all $i \in C$ and $j \notin C$,*
$u_i(\acute{\sigma}_C, \rho) \leq u_i(D)$, and
$u_j(\acute{\sigma}_C, \acute{\rho}) \leq u_j(\acute{\sigma}_C, \rho)$,
where $\acute{\sigma}_C$ satisfies the condition $u_i(D) < u_i(\acute{\sigma}_C, \rho'')$ for some $i \in C$ and some strategy vector ρ'' for $-C$.

If there is a punishment strategy that satisfies this definition, when players abort in the cheap talk and declare actions which they will take in order to receive more payoff than correlated equilibrium D, for the rest of players, taking a punishment strategy is better than following their temptation.

6 Conclusion

We showed that there are cases when a punishment strategy does not work. We suggested a new suggested new definition of punishment strategy to avoid the cases.

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$P_1\ a^1_1$

$P_2\ a^1_2$

$\downarrow P_5$	P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_5	a^1_4	(4,4,3,3,3)	(4,4,3,4,3)	(5,5,5,5,5)	(3,4,4,5,4)
	a^2_4	(6,5,3,5,5)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
	a^3_4	(3,6,4,4,4)	(3,4,3,4,4)	(6,4,3,4,4)	(3,3,3,3,3)
	a^4_4	(4,3,3,3,3)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^2_5	a^1_4	(4,3,3,3,3)	(4,4,3,3,3)	(6,4,4,5,5)	(3,4,4,4,4)
	a^2_4	(3,5,4,5,5)	(6,5,4,3,3)	(4,5,3,4,4)	(3,4,4,4,4)
	a^3_4	(3,6,5,4,4)	(3,4,3,3,3)	(4,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(3,3,5,4,3)	(3,4,4,3,3)	(4,3,4,4,4)	(4,3,4,4,4)
a^3_5	a^1_4	(4,4,3,4,3)	(6,4,4,4,4)	(4,4,3,3,5)	(3,3,3,3,3)
	a^2_4	(6,5,4,3,3)	(3,4,5,4,5)	(5,5,4,4,3)	(3,4,4,5,4)
	a^3_4	(3,4,3,4,4)	(6,4,4,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(5,3,4,4,4)	(3,5,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^4_5	a^1_4	(5,4,5,4,4)	(4,4,4,3,3)	(4,3,4,5,4)	(3,4,4,4,4)
	a^2_4	(4,3,4,4,5)	(6,5,4,5,4)	(4,3,4,4,4)	(3,4,4,4,4)
	a^3_4	(4,6,4,5,5)	(3,5,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)

$P_1\ a^2_1$

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,5,3,3)	(4,4,3,4,4)	(6,4,4,4,4)	(3,4,4,5,4)
a^2_4	(4,6,4,4,4)	(6,5,4,3,3)	(3,4,4,4,5)	(3,4,4,5,4)
a^3_4	(6,6,4,5,4)	(4,4,4,3,3)	(6,4,4,3,3)	(3,3,3,3,3)
a^4_4	(4,6,4,4,4)	(3,3,3,3,3)	(3,5,3,3,3)	(3,3,3,3,3)
a^1_4	(4,6,4,4,4)	(4,3,4,4,4)	(4,6,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,6,4,4)	(5,5,4,4,4)	(6,5,4,3,3)	(4,4,4,4,4)
a^3_4	(6,4,4,6,3)	(4,5,3,4,4)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,5,3,4,4)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,3,4,3)	(4,4,3,4,3)	(4,5,3,4,4)	(3,3,3,3,3)
a^2_4	(6,5,4,3,3)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,5,4)
a^3_4	(3,6,3,4,3)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(5,3,4,4,4)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(5,3,4,4,4)	(4,4,4,3,3)	(3,4,4,4,4)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(4,5,4,5,3)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)

$P_1\ a^3_1$

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(6,4,5,4,4)	(4,4,3,4,3)	(4,4,4,4,4)	(3,3,3,3,3)
a^2_4	(4,3,4,5,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,5,4)
a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(6,4,5,4,4)	(4,4,4,3,3)	(4,4,4,4,4)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(4,4,4,5,5)	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)
a^1_4	(4,4,3,4,3)	(4,5,6,4,4,)	(4,4,3,3,5)	(4,4,4,3,3)
a^2_4	(6,5,4,3,3)	(4,4,3,4,4)	(5,5,4,4,3)	(6,5,4,4,3)
a^3_4	(3,4,3,4,4)	(4,4,3,4,4)	(3,4,3,4,4)	(3,4,3,3,3)
a^4_4	(5,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)
a^1_4	(4,3,4,4,4)	(4,4,3,3,3)	(5,3,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(4,4,4,4,4)
a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,5,5,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,4,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)

$P_1\ a^4_1$

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,3,4,5,5)	(4,4,3,3,5)	(5,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,5,5)	(5,5,4,4,3)	(4,4,3,3,4)	(3,3,3,4,3)
a^3_4	(4,3,5,4,4)	(3,4,3,4,4)	(4,4,3,4,4)	(3,3,3,3,3)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(3,5,3,3,3)	(3,3,3,3,3)
a^1_4	(4,4,4,3,3)	(4,4,4,4,4)	(6,4,5,4,4,)	(3,4,4,4,4)
a^2_4	(6,5,4,4,3)	(4,4,3,4,4)	(4,3,4,4,4)	(3,4,4,4,4)
a^3_4	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,5,5)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,3,3,5)	(4,4,4,3,3)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(6,5,4,4,3)	(6,5,4,3,3)	(3,4,4,4,4)
a^3_4	(3,4,3,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

$P_2\ a^2_2$

$\downarrow P_5$	P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_5	a^1_4	(6,4,5,4,4)	(4,4,3,4,3)	(4,5,6,4,4)	(3,3,3,3,3)
	a^2_4	(4,3,4,5,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,5,4)
	a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^2_5	a^1_4	(6,4,5,4,4)	(4,4,4,3,3)	(4,4,4,4,4)	(3,4,4,4,4)
	a^2_4	(4,3,4,4,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,4,4)
	a^3_4	(4,4,4,5,5)	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)
a^3_5	a^1_4	(4,4,3,4,3)	(4,5,6,4,4,)	(4,4,3,3,5)	(4,4,4,3,3)
	a^2_4	(6,5,4,3,3,)	(4,4,3,4,4)	(5,5,4,4,3)	(6,5,4,4,3)
	a^3_4	(3,4,3,4,4)	(4,4,3,4,4)	(3,4,3,4,4)	(3,4,3,3,3)
	a^4_4	(5,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)
a^4_5	a^1_4	(4,3,4,4,4)	(4,4,3,3,3)	(5,3,3,4,3)	(3,4,4,4,4)
	a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(4,4,4,4,4)
	a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,5,5,4)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,4,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,3,3)	(4,4,3,4,3)	(3,4,3,4,4)	(3,4,4,5,4)
a^2_4	(6,5,3,5,5)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(3,6,4,4,4)	(3,4,3,3,3)	(4,4,3,4,4)	(3,3,3,3,3)
a^4_4	(4,3,3,3,3)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,3,3,3)	(4,4,3,3,3)	(6,4,4,5,5)	(3,4,4,4,4)
a^2_4	(3,5,4,5,5)	(6,5,4,3,3)	(4,5,3,4,4)	(3,4,4,4,4)
a^3_4	(3,6,4,4,4)	(5,5,5,5,5)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(3,3,5,4,3)	(3,4,4,3,3)	(4,3,4,4,4)	(4,3,4,4,4)
a^1_4	(4,4,3,4,3)	(6,4,4,4,4)	(4,4,3,3,5)	(3,3,3,3,3)
a^2_4	(6,5,4,3,3,)	(3,4,5,4,5)	(5,5,4,4,3)	(3,4,4,5,4)
a^3_4	(3,4,3,4,4)	(6,4,4,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(5,3,4,4,4)	(3,5,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(5,4,5,4,4)	(4,4,4,3,3)	(4,3,4,5,4)	(3,4,4,4,4)
a^2_4	(4,3,4,4,5)	(6,5,4,5,4)	(4,3,4,4,4)	(3,4,4,4,4)
a^3_4	(4,6,4,5,5)	(3,5,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,3,3)	(4,4,3,4,4)	(6,4,4,4,4)	(3,4,4,5,4)
a^2_4	(4,6,4,4,4)	(6,5,4,3,3)	(3,4,5,4,5)	(3,4,4,5,4)
a^3_4	(4,3,4,5,5)	(4,4,5,3,3)	(6,4,4,3,3)	(3,3,3,3,3)
a^4_4	(4,6,4,4,4)	(3,3,3,3,3)	(3,5,3,3,3)	(3,3,3,3,3)
a^1_4	(4,6,4,4,4)	(4,3,4,4,4)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,6,4,4,)	(5,5,4,4,4)	(6,5,4,3,3)	(4,4,4,4,4)
a^3_4	(3,4,4,3,4,)	(4,5,3,4,4)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,5,3,4,4)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,3,4,3)	(4,4,3,4,3)	(4,5,6,4,4,)	(3,3,3,3,3)
a^2_4	(6,5,4,3,3)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,5,4)
a^3_4	(3,4,3,4,4)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(5,3,4,4,4)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(5,3,4,4,4)	(4,4,4,3,3)	(5,5,5,5,5)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,3,4,4)	(3,3,3,3,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(3,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,3,5)	(5,4,3,4,3)	(6,4,5,4,4,)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(4,4,3,3,4)	(4,3,4,4,4)	(3,3,3,4,3)
a^3_4	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,5,5)	(3,3,3,3,3)
a^4_4	(4,3,4,4,4)	(3,5,3,3,3)	(4,3,4,4,4)	(3,3,3,3,3)
a^1_4	(4,3,4,5,5)	(4,4,4,4,4)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,5,5)	(4,4,3,4,4)	(6,5,4,3,3,)	(3,4,4,4,4)
a^3_4	(4,3,5,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,4,3,3)	(4,4,3,3,5)	(4,4,4,3,3)	(3,4,4,4,4)
a^2_4	(6,5,4,4,3)	(5,5,4,4,3)	(6,5,4,4,3)	(3,4,4,4,4)
a^3_4	(3,4,3,3,3)	(3,4,3,4,4)	(3,4,3,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

Figure 2: An example of game that a punishment strategy does not work(1)

P_1 a^1_1
P_2 a^3_2

↓ P_5	P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_5	a^1_4	(4,4,3,3,3)	(4,4,3,4,4)	(4,4,4,4,4)	(3,4,4,5,4)
	a^2_4	(4,6,4,4,4)	(6,5,4,3,3)	(3,4,5,4,5)	(3,4,4,5,4)
	a^3_4	(4,3,4,5,5)	(4,4,5,3,3)	(6,4,4,3,3)	(3,3,3,3,3)
	a^4_4	(4,6,4,4,4)	(3,3,3,3,3)	(3,5,3,3,3)	(3,3,3,3,3)
a^2_5	a^1_4	(4,6,4,4,4)	(4,3,4,4,4)	(4,4,3,4,3)	(3,4,4,4,4)
	a^2_4	(4,3,6,4,4)	(5,5,4,4,4)	(6,5,4,3,3)	(4,4,4,4,4)
	a^3_4	(3,4,4,3,4)	(4,5,3,4,4)	(3,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,5,3,4,4)	(5,3,4,4,4)	(4,4,4,4,4)
a^3_5	a^1_4	(4,4,3,4,3)	(4,4,3,4,3)	(4,5,6,4,4,)	(3,3,3,3,3)
	a^2_4	(6,5,4,3,3)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,5,4)
	a^3_4	(3,4,3,4,4)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(5,3,4,4,4)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^4_5	a^1_4	(5,3,4,4,4)	(4,4,4,3,3)	(5,5,5,5,5)	(3,4,4,4,4)
	a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
	a^3_4	(4,4,4,5,5)	(3,4,3,4,4)	(4,4,3,4,4)	(3,3,3,3,4)
	a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(3,4,4,4,4)

P_1 a^2_1

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(6,4,5,4,4)	(4,4,3,4,3)	(4,5,6,4,4)	(3,3,3,3,3)
a^2_4	(4,3,4,5,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,5,4)
a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(6,4,5,4,4)	(4,4,4,3,3)	(4,4,4,4,4)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,4,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(4,4,4,5,5)	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)
a^1_4	(4,4,3,4,3)	(4,5,6,4,4)	(4,4,3,3,5)	(4,4,4,3,3)
a^2_4	(6,5,4,3,3)	(4,4,3,4,4)	(5,5,4,4,3)	(6,5,4,4,3)
a^3_4	(3,4,3,4,4)	(4,4,3,4,4)	(3,4,3,4,4)	(3,4,3,3,3)
a^4_4	(5,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)
a^1_4	(4,3,4,4,4)	(4,4,3,3,3)	(5,3,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,4,4)	(6,5,4,3,3)	(4,4,3,4,4)	(4,4,4,4,4)
a^3_4	(4,6,4,5,5)	(3,4,3,4,4)	(4,4,5,5,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,4,4,4,4)	(4,3,3,3,3)	(4,4,4,4,4)

P_1 a^3_1

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,3,3)	(4,4,3,4,3)	(4,4,3,4,4)	(3,4,4,5,4)
a^2_4	(6,5,3,5,5)	(6,5,4,3,3)	(4,4,3,4,4)	(3,4,4,4,4)
a^3_4	(3,6,4,4,4)	(3,4,3,4,4)	(4,3,5,3,3)	(3,3,3,3,3)
a^4_4	(4,3,3,3,3)	(5,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,3,3,3)	(4,4,3,3,3)	(6,4,4,5,5)	(3,4,4,4,4)
a^2_4	(3,5,4,5,5)	(6,5,4,3,3)	(4,5,3,4,4)	(3,4,4,4,4)
a^3_4	(3,6,5,4,4)	(3,4,3,3,3)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	**(5,5,5,5,5)**	(3,4,4,3,3)	(4,3,4,4,4)	(4,3,4,4,4)
a^1_4	(4,4,3,4,3)	(6,4,4,4,4)	(4,4,3,3,5)	(3,3,3,3,3)
a^2_4	(6,5,4,3,3)	(3,4,5,4,5)	(5,5,4,4,3)	(3,4,4,5,4)
a^3_4	(3,4,3,4,4)	(6,4,4,3,3)	(5,6,3,4,3)	(4,4,4,4,4)
a^4_4	(5,3,4,4,4)	(3,5,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(5,4,5,4,4)	(4,4,4,3,3)	(4,3,4,5,4)	(3,4,4,4,4)
a^2_4	(4,3,4,4,5)	(6,5,4,5,4)	(4,3,4,4,4)	(3,4,4,4,4)
a^3_4	(4,6,4,5,5)	(3,5,3,4,4)	(4,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)

P_1 a^4_1

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(5,4,3,4,3)	(6,4,5,4,4,)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,4,3,3,4)	(4,3,4,4,4)	(6,5,4,3,3,)	(3,3,3,4,3)
a^3_4	(4,4,3,4,4)	(4,4,4,5,5)	(3,4,3,4,4)	(3,3,3,3,3)
a^4_4	(3,5,3,3,3)	(4,3,4,4,4)	(5,3,4,4,4)	(3,3,3,3,3)
a^1_4	(4,4,3,3,5)	(4,4,4,4,4)	(4,4,4,3,3)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(4,4,3,4,4)	(6,5,4,4,3)	(3,4,4,4,4)
a^3_4	(3,4,3,4,4)	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,4,5,5)	(4,4,4,3,3)	(4,4,3,3,5)	(3,4,4,4,4)
a^2_4	(4,3,4,5,5)	(6,5,4,4,3)	(5,5,4,4,3)	(3,4,4,4,4)
a^3_4	(4,3,5,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

P_2 a^4_2

↓ P_5	P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_5	a^1_4	(5,4,3,4,3)	(6,4,5,4,4)	(4,4,3,4,3)	(3,4,4,4,4)
	a^2_4	(4,4,3,3,4)	(4,3,4,4,4)	(6,5,4,3,3)	(3,3,3,4,3)
	a^3_4	(4,4,3,4,4)	(4,4,4,5,5)	(3,4,3,4,4)	(3,3,3,3,3)
	a^4_4	(3,5,3,3,3)	(4,3,4,4,4)	(5,3,4,4,4)	(3,3,3,3,3)
a^2_5	a^1_4	(4,4,3,3,5)	(4,4,4,4,4)	(4,4,4,3,3)	(3,4,4,4,4)
	a^2_4	(5,5,4,4,3)	(4,4,3,4,4)	(6,5,4,4,3)	(3,4,4,4,4)
	a^3_4	(3,4,3,4,4)	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^3_5	a^1_4	(4,3,4,5,5)	(4,4,4,3,3)	(4,4,3,3,5)	(3,4,4,4,4)
	a^2_4	(4,3,4,5,5)	(6,5,4,4,3)	(5,5,4,4,3)	(3,4,4,4,4)
	a^3_4	(4,3,5,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^4_5	a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
	a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
	a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
	a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,3,5)	(5,4,3,4,3)	(6,4,5,4,4)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(4,4,3,3,4)	(4,3,4,4,4)	(3,3,3,4,3)
a^3_4	(3,4,3,4,4)	(4,4,3,4,4)	(4,4,4,5,5)	(3,3,3,3,3)
a^4_4	(4,3,4,4,4)	(3,5,3,3,3)	(4,3,4,4,4)	(3,3,3,3,3)
a^1_4	(4,3,4,5,5)	(4,4,4,4,4)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,5,5)	(4,4,3,4,4)	(6,5,4,3,3)	(3,4,4,4,4)
a^3_4	(4,3,5,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,4,3,3)	(4,4,3,3,5)	(4,4,4,3,3)	(3,4,4,4,4)
a^2_4	(6,5,4,4,3)	(5,5,4,4,3)	(6,5,4,4,3)	(3,4,4,4,4)
a^3_4	(3,4,3,3,3)	(3,4,3,4,4)	(3,4,3,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,3,4,5,5)	(4,4,3,3,5)	(5,4,3,4,3)	(3,4,4,4,4)
a^2_4	(4,3,4,5,5)	(5,5,4,4,3)	(4,4,3,3,4)	(3,3,3,4,3)
a^3_4	(4,3,5,4,4)	(3,4,3,4,4)	(4,4,3,4,4)	(3,3,3,3,3)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(3,5,3,3,3)	(3,3,3,3,3)
a^1_4	(4,4,4,3,3)	(4,4,4,4,4)	(6,4,5,4,4)	(3,4,4,4,4)
a^2_4	(6,5,4,4,3)	(4,4,3,4,4)	(4,3,4,4,4)	(3,4,4,4,4)
a^3_4	(3,4,3,3,3)	(3,4,3,3,3)	(4,4,4,5,5)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,3,3,5)	(4,4,4,3,3)	(4,4,3,4,3)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(6,5,4,4,3)	(6,5,4,3,3)	(3,4,4,4,4)
a^3_4	(3,4,3,4,4)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(5,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

P_3/P_4	a^1_3	a^2_3	a^3_3	a^4_3
a^1_4	(4,4,3,4,3)	(4,4,3,5,5)	(5,4,3,4,3)	(3,4,4,4,4)
a^2_4	(6,5,4,3,3)	(5,5,4,3,3)	(4,4,3,3,4)	(3,3,3,4,3)
a^3_4	(3,4,3,4,4)	(3,4,3,4,4)	(4,4,3,4,4)	(3,3,3,3,3)
a^4_4	(5,3,4,4,4)	(3,4,3,4,4)	(3,5,3,3,3)	(3,3,3,3,3)
a^1_4	(4,4,4,3,3)	(4,4,4,4,4)	(4,4,3,3,5)	(3,4,4,4,4)
a^2_4	(6,5,4,4,3)	(4,4,3,4,4)	(5,5,4,4,3)	(3,4,4,4,4)
a^3_4	(3,4,3,3,3)	(3,4,3,3,3)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,3,3,3)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,4,3,3,5)	(6,4,5,4,4,)	(4,4,3,3,5)	(3,4,4,4,4)
a^2_4	(5,5,4,4,3)	(4,3,4,4,4)	(5,5,4,4,3)	(3,4,4,4,4)
a^3_4	(3,4,3,4,4)	(4,4,4,5,5)	(3,4,3,4,4)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(4,3,4,4,4)	(4,3,4,4,4)	(4,4,4,4,4)
a^1_4	(4,3,5,4,4)	(4,4,3,5,5)	(5,4,3,3,3)	(4,4,4,4,4)
a^2_4	(4,3,4,4,4)	(5,5,4,3,3)	(4,4,3,5,5)	(4,4,4,4,4)
a^3_4	(4,4,5,4,3)	(3,4,3,4,4)	(4,3,4,3,3)	(4,4,4,4,4)
a^4_4	(4,3,4,4,4)	(3,4,3,4,4)	(4,6,3,4,5)	(4,4,4,4,4)

Figure 3: An example of game that a punishment strategy does not work(2)